6. Deformation Analysis

Deformation analysis is the study of the changes the shape of an arbitrary body in time. In surveying it has come to mean the observations of the geometrical changes of a body. Thus it might concern the study of dams, tectonic movements etc. The approach must depend on the desired result and the accessible measuring devices. For surveying the most important approach is through geodetic or photogrammetric methods, where observation data from different epochs are compared. Usually the analysis is divided in three types depending on its characteristics.

- Deformation of static bodies
- Deformation of kinematic bodies
- Deformation of dynamic bodies

The deformation of static bodies is where the movement vector between two or more epochs is the interesting result. The deformation of kinematic bodies is when one is interested of a function that describes the movement of a body, e.g. to describe the movement of a body through its velocity and acceleration. The deformation of dynamic bodies is normally used for processes that change form during a period of time and return to the original state afterwards, e.g. the deformation of a bridge while a car goes across it. This thesis will concentrate on the static bodies, as those with available data are the most adequate for the problem of Santiago de Puriscal. If more epochs had been measured a kinematic study would have been in place.

The adjustment of the observations of a deformed body can be described by the following matrix model. This model is for the general case with n epochs.

$$L + V = \begin{matrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & & \\ \cdots & & \cdots & \\ A_{n1} & & & A_{nn} \end{matrix} * \begin{matrix} x_1 \\ \cdots \\ \\ x_n \end{matrix} \qquad (6.1)$$

In this model L are the observations, V the residuals, A_{11} to A_{nn} are the configuration matrixes of the observation of each epoch and A_{ij} are the functional connection between the observations of the epochs. In some special cases the functional connection can be of interest, but generally there is no functional connection between the epochs or it is ignored. Thus A_{ij} $(i \neq j)$ is considered a zero matrix. Therefore the following simplification can be done (Pelzer, 1985).

$$L+V=\begin{vmatrix} A_{11} & 0 & \dots & 0 \\ 0 & A_{22} & & \\ \dots & & \dots & \\ 0 & & & A_{nn} \end{vmatrix} * \begin{vmatrix} x_1 \\ \dots \\ \\ x_n \end{vmatrix} \qquad (6.2)$$

In the above model the problem will be to prove that the x_1 to x_n are equal or not. The process of how to prove this and how to locate the displaced points will be described in the following chapters. This process of proving may seem unnecessary in cases where the movement in all points is considerably bigger than the standard error. However, in cases where the size of the displacements are close to the standard error of the observations such a test is in place. The model for a deformation analysis of two epochs is as follows.

$$L+V=\begin{vmatrix} A_{11} & 0 \\ 0 & A_{22} \end{vmatrix} * \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \qquad (6.3)$$

The above described methods only detect the displacements of a body, but do not describe the movement as a function of time or other factors. It is possible to build a function where the objective would for example be to analyse the velocity and acceleration of the body. A simple mathematical expression of a kinematic body could thus, by the help of Taylor series, be.

$$\begin{aligned} x_2 &= x_1 + \frac{\partial x}{\partial t}(t_2 - t_1) + \frac{1}{2}\frac{\partial^2 x}{\partial t^2}(t_2 - t_1)^2 + \dots \\ &= x_1 + \dot{x}\Delta t + \frac{1}{2}\ddot{x}\Delta t^2 + \dots \end{aligned} \qquad (6.4)$$

This formula can now in a rather simple way be transferred to a matrix form, where the sought parameters are $x, \dot{x}$ and $\ddot{x}$.

$$\begin{vmatrix} L_1 \\ L_2 \\ L_3 \\ \dots \\ L_k \end{vmatrix} + \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ \dots \\ V_k \end{vmatrix} = \begin{vmatrix} A_1 & A_1\Delta t & A_1\Delta t^2 \\ A_2 & A_2\Delta t & A_2\Delta t^2 \\ A_3 & A_3\Delta t & A_3\Delta t^2 \\ \dots & & \dots \\ A_k & A_k\dot{\Delta} t & A_k\Delta t^2 \end{vmatrix} * \begin{vmatrix} x \\ \dot{x} \\ \ddot{x} \end{vmatrix} \qquad (6.5)\ \text{(Pelzer, 1985)}$$

Here A_i is the configuration matrix, Δt the time difference, x the displacement and $\dot{x}$, $\ddot{x}$ the velocity and the acceleration respectively. It is thus possible to build a function, from the observation data, to describe the needed physical phenomenon. The adjustment or treatment of kinematic analysis is beyond the objective of this thesis.

6.1. Absolute and Relative Deformation observations

The deformed body can be observed with some known and stable external points. Then the observation type is called absolute deformation observations. In the absolute deformation network the displacement of a body is compared to its stable neighbourhood. Thus it is possible to calculate the velocity of a body, e.g. in Santiago de Puriscal it would be possible to find how fast the landslide is moving.

If no points outside the deforming body can be considered stable, the network must be considered as relative. In the relative network the internal difference (between observations or co-ordinates) between the epochs is the interesting one. It will only be possible to detect changes in the geometry of an network and not whether the network as a whole is moving or rotating. This results in the fact that in the Puriscal network only internal strain can be found.

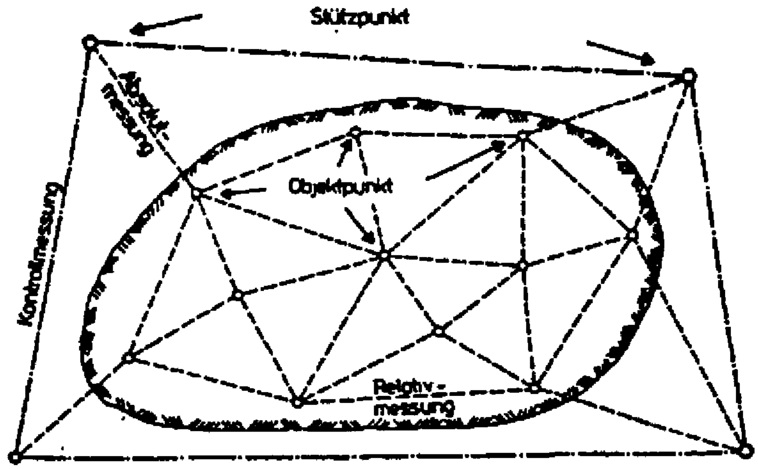

Figure 6.1. Absolute and relative network (Pelzer, 1985).

In both relative and absolute networks datum defects can be found. The problem is to decide whether the detected displacements are the result of real movements. They can also result from errors in the observations or differences in datum of the two epochs.

In the case of absolute observations the equation (6.2) can be modified by excluding the stable points from the moving ones. Then the following equation is obtained.

$$L+V=\begin{vmatrix} A_{s1} & A_{11} & 0 & \dots & 0 \\ A_{s2} & 0 & A_{22} & & \\ \dots & & & \dots & \\ A_{sn} & 0 & \dots & & A_{nn} \end{vmatrix} * \begin{vmatrix} x_s \\ x_1 \\ \dots \\ x_n \end{vmatrix} \qquad (6.6)$$

In this equation A_{Sj} describes the configuration matrixes of the stable points that now define the datum, x_s is thus the vector of unknowns for stable points. In this case the other points, the unstable ones are said to be the object points. For a relative analysis the general equation (6.2) is valid.

6.2. The Datum problem

When discussing the datum problem a distinction between absolute and relative networks must be done, as the problem of datum defect (see chapter 6.2.1) is different. In the absolute network the datum is defined by stable points, but it should be tested whether they have moved or not between the epochs. In a relative network the datum is more difficult to define as it is not possible to assume any stability, i.e. all points can move in any direction. Thus there is no configuration of points that can be used as a datum without a prior testing. It is even questionable whether a datum is interesting in a relative network at all. In many cases the desired result is not relative to the datum but the differences between the observations in the epochs. The difference between two distances does not require any datum (if the length unit is not considered a datum).

Dimen -sion	Type of Network	Type of observations	datum defects	Free datum parameters
1	Height network	Levelling	1	1 Translation
2	Plane network	Distances, Azimuths and Directions	4	2 Translations 1 Rotation 1 Scaling
3	Space network	Distances, Azimuths, Zenith Azimuths and Directions	7	3 Translations 3 Rotations 1 Scaling

Table 6.1. Free datum parameters.

6.2.1. Datum defect

The datum defect should be understood as any kind of differences that are not deformation dependant nor in the datum definition between the involved epochs. For example the different scale of the networks in various epochs is then taken care of. Due to the free network adjustment it might as well be necessary to translate and rotate the network. Table 6.1 shows which parameters define the datum defect.

6.3. The S-Transformation

In 1973 Baarda developed a method that allows transformations of networks between datums. This method is called S-transformation (Similarity Transformation), which is used to transform all epochs to a common datum. With the S-transformation it is possible to transform both the displacement vector and the covariance matrix to a common datum. Even though the adjustment is done in an arbitrary datum. With this method some datum defects can be eliminated, e.g. the scale error, rotation or the translation between epochs.

To do the S-transformation from the datum *k* to the datum *i*, an S-matrix is built by

$$S_i = I - G(G^t E_i G)^{-1} G^t E_i \quad (6.7)$$

where I is the unity matrix and G is a configuration matrix for the free datum parameters. E is a diagonal matrix that decides which points are datum defining. If a point is datum defining the element in the diagonal is 1, otherwise 0. The G-matrix can have the following form.

$$G = \begin{vmatrix} 1 & 0 & y_1 & x_1 \\ 0 & 1 & -x_1 & y_1 \\ 1 & 0 & y_2 & x_2 \\ 0 & 1 & -x_2 & y_2 \\ . & . & . & . \\ . & . & . & . \\ 1 & 0 & y_n & x_n \\ 0 & 1 & -x_n & y_n \end{vmatrix} \quad (6.8)$$

where x and y are reduced with respect to the centroid[8] of the network. This G-matrix is an example with two translations and one rotation, comparable to the matrix configuration of a Helmert transformation in 2 dimensions. The G-matrix can be constructed to fulfil the needs of each application.

The advantage of the S-transformation is that it does not change the inner geometry of the network and it is not necessary to know the datum one wants to transform from. With the S-matrix it is possible to transform both the co-ordinate vector x_k and the covariance matrixes Q_k to a common datum by

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[8]Reduction with respect to the centriod : ${}_{reduced}X_i = X_i - (1/_n)\Sigma X_i$. For Y and Z in the same way.

$$x_i = S_i x_k \qquad (6.9)$$

$$Q_i = S_i Q_k S_i^t \qquad (6.10)$$

6.4. Free network adjustment

Free adjustment is done to avoid all strain that occur in the network when known co-ordinates constrain the residuals. Here three methods of doing free network adjustment will be discussed.

- Strain-free adjustment
- Total trace minimisation
- Part trace minimisation

6.4.1. Strain-free adjustment

In the Strain-free adjustment two points are used as fixed, they then define the datum. Thus no strain is evaluated in the network, as the two co-ordinates only define orientation and scale of the network. An advantage of this method is that it can be used in any adjustment program. A disadvantage is that the two known points are errorless. The effect of the strain-free adjustment can be seen in the following figure.

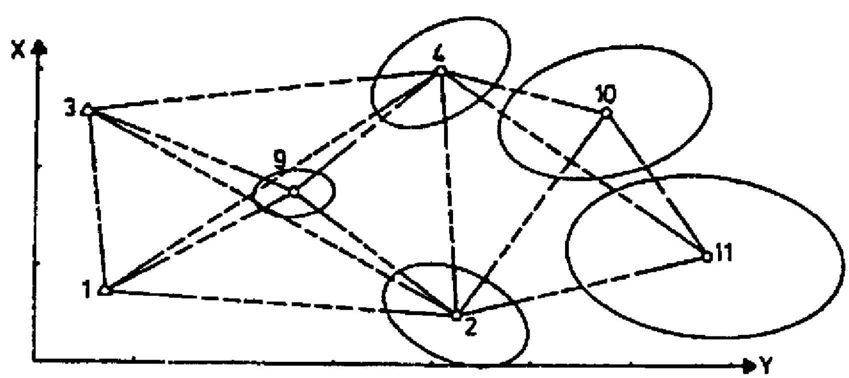

Figure 6.2. Strain free adjustment. Δ is for the fixed points, 1 and 3. o indicates the new points. The ellipses indicate the error ellipses (Pelzer 1985).

6.4.2. Total trace minimisation[9]

In the Total trace minimisation all points define the datum and no point is fixed. The advantage of this method is that no point is errorless. The disadvantage is that a special adjustment program is needed. The datum in this kind of network adjustment is the centroid of the network. In this type of adjustment all points have an error.

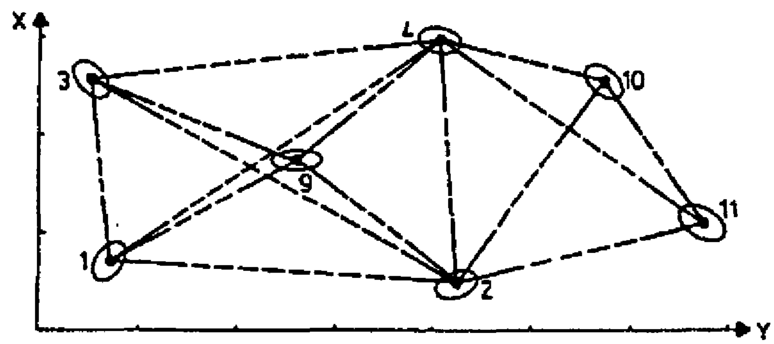

Figure 6.3. Total trace minimisation. • indicates the datum defining points. The ellipses indicate the error ellipses (Pelzer 1985).

6.4.3. Part trace minimisation[10]

In the Part trace minimisation some points define the datum and the rest are new points. The way of calculating is almost the same as for the Total trace minimisation. For deformation studies this type of adjustment is the most suited one, as the points can be divided into two groups. One for those that define the datum and other for the new points, i.e. stable and object points.

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[9]Direct translation from the German term "Gesamtspurminimierung", which means a minimisation of the whole trace.

[10]Direct translation from the German term "Teilspurminimierung", which means a minimisation of a part of the trace.

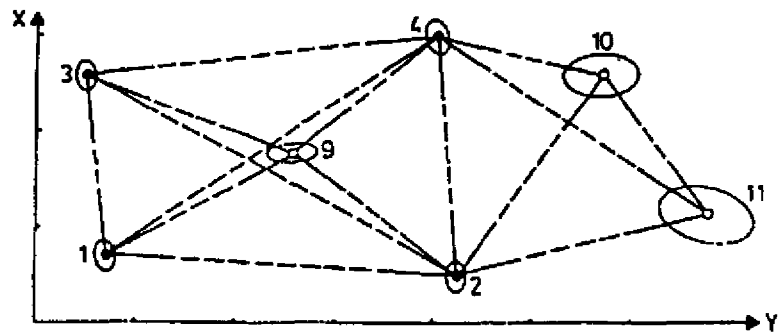

Figure 6.4. Part trace minimisation. • indicates the datum defining points. o indicates the new points. The ellipses indicate the error ellipses (Pelzer 1985).

To calculate the total trace and the part trace minimisation the following technique is used.

$$\begin{vmatrix} A^tQ_{ll}^{-1}A & G_d^t \\ G_d & 0 \end{vmatrix}^{-1} = \begin{vmatrix} Q_{xx} & G_d(G_d^tG_d)^{-1} \\ (G_d^tG_d)^{-1}G_d & 0 \end{vmatrix} \Leftrightarrow G_d = E_iG \quad (6.11)$$

where G is the same as the G-matrix in formula 6.7, and E_i is a diagonal matrix with one for elements defining the datum and zero for new points. It is advantageous to include the G-matrix, as orthogonal bordering in the matrix inversion, as $(A^tQ_{ll}^{-1}A)$ for a free network is singular and has therefore no unique solution. The condition $(A^tQ_{ll}^{-1}A)G_d = 0$ must be fulfilled. Q_{ll}^{-1} is the weight matrix of the observations. In this thesis the question of how this matrix inverse is obtained will not be discussed, but we refer to literature on generalised matrix inverses, e.g. Sjöberg (1990).

6.5. Statistical testing of a deformation model

Consider the following matrix equation describing a constraint on the deformation :

$$Bx = w \quad (6.12)$$

where B describes the movement, x is the co-ordinate vector and w the misclosure

We now define a null hypothesis (H_0) that no deformation has taken place. The alternative hypothesis (H_a) is that deformation has occurred.

$$H_0 : Bx = 0 \qquad (6.13)$$

$$H_a : Bx <> 0 \qquad (6.14)$$

Those two hypotheses are tested with some appropriate statistical test, e.g. the Global Congruency test, see below.

6.6. The Global Congruency Test[11]

In order to judge if any movement within a certain network is statistically secure, some kind of statistic test has to be performed. The Global Congruency Test is the one most commonly used. In this test a deformation related parameter is calculated from the data and compared to its theoretical distribution.

The displacement between two epochs can be written as

$$d = x_2 - x_1 \quad (6.15)$$

where x_1, x_2 are the co-ordinate vectors of each epoch. In this case we form (6.12) with B= [-I, I] , $x = |x_1 \quad x_2|^t$ and d=w, where I is the unit matrix.

Now the null hypothesis $H_0 : d = 0$ can be tested against the alternative hypothesis $H_a : Bx <> 0$.

Assuming that the estimated vectors x_1 and x_2 are uncorrelated and have the covariance matrixes Q_{xx1} and Q_{xx2} , the covariance matrix of the displacement d becomes

$$Q_{dd} = Q_{xx1} + Q_{xx2} \quad (6.16)$$

Furthermore, the standard error of d becomes

$$s_0 = \sqrt{\frac{(v^t pv)_1 + (v^t pv)_2}{f}} \quad (6.17)$$

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[11]Direct translation from the German term "Globales Kongruenz Test".

$$f = f_1 + f_2 \quad (6.18)$$

$(v^t pv)_i$ is calculated as in chapter 4 and f_i is the number of degrees of freedom. To see whether a displacement has taken place, the following test quantity is calculated

$$F = \frac{d\, Q_{dd}^{-1} d'}{s_0\, f} \qquad (6.19)\ (\text{Pelzer, } 1985)$$

This value is compared to a value determined by the F-distribution (see chapter 6.6.1). If F is bigger than this value then the displacement can be considered non zero or significant i.e. d<>0. Otherwise if F is less than this value then the null hypothesis H_0 is judged true, i.e. no displacement within the network has taken place.

6.6.1. The F-distribution

The F-distribution or the Fisher distribution was developed by R.A. Fisher in 1924. It is based on the χ^2-distribution. It is used to compare the distribution of two different standard errors, m and n. It is defined as follows.

$$F_{m,n} = \frac{\dfrac{\chi_m^2}{f_m}}{\dfrac{\chi_n^2}{f_n}} \qquad (6.20)$$

where f_m and f_n are the number of degrees of freedom of χ_m^2 and χ_n^2. As the definition of the χ^2-distribution is

$$\chi^2 = \frac{f_i * s_i^2}{\sigma_i^2} \qquad (6.21)$$

where f is the number of degrees of freedom, σ is the theoretical standard error and s the empirical one. If (6.21) is put into (6.20), the following relation between two variances is obtained.

$$F_{m,n} = \frac{\sigma_2^2}{\sigma_1^2} \cdot \frac{s_1^2}{s_2^2} \qquad (6.22)$$

where m,n are the number of degrees of freedom for each variance. For further information on the F-distribution we refer to Pelzer (1985).

6.7. Localisation of displaced points

There exist various methods to find if a points is significantly moved. The ones that will be described here are the method of maximal displacements, S-transformation and an approximate analysis.

6.7.1. The method of maximal displacements[12]

The method of maximal displacements divides all the points in two parts; stable and displaced, where the stable points define the datum. The way of testing is the following: remove points with significant displacement from the stable points and keep the rest as stable points. The test value is obtained as follows

$$R_i = d_i^t (Q_{dd}^{-1})_i \, d_i \qquad (6.23)$$

where d, Q are defined as in chapter 6.6. R_{max} is then eliminated from the stable points. The stable points are transformed to a common datum, and a Global Congruency test is done to control whether there is any significant movement left in the network. Then the process is repeated until the Global Congruency test shows no significant movements within remaining datum points. Then the points remaining stable have not had any significant movement while the ones in the displaced group have moved. Below the process is shown in schematic form.

1) Global Congruency test to test significant displacement
2) Calculate R_i for all stable points
3) Put the point with biggest R_i among the displaced ones
4) Through an S-transform (see chapter 6.3) the remaining stable points are transformed to a common datum
5) Repeat the process until the Global Congruency test shows no significant displacement left in the network

It is important that the points participating in the test exist in both epochs, as they in one epoch have no co-ordinates nor any variances. Thus this test is ideal for identical network configurations.

[12]Direct translation of the German term "Maximalen Klaffungsanteile", which means maximal displacement parts.

6.7.2. By S-transformation

Localisation with S-transformations is done by transforming both epochs to a common datum by S-transformations, as described in chapter 6.3. The method divides the points in three groups; stable, displaced and not common in both epochs. First d_f and Q_{ff} are calculated as follows.

$$d_f = (x_f)_2 - (x_f)_1 \qquad (6.24)$$

$$Q_{ff} = (Q_{ff})_1 + (Q_{ff})_2 \qquad (6.25)$$

$$R_f = d_f^t Q_{ff}^{-1} d_f \qquad (6.26)$$

where the index outside the parenthesis indicate the epoch and f indicates fixed points. R_f is calculated, as in the global congruency test, for all combinations of all but one fixed point. Then the point with minimum R_f is significantly displaced. Below the process is shown in schematic form.

1) Consider all common points as fixed.
2) S-transform both networks to a common datum.
3) Calculate d_f and Q_{ff}
4) Calculate R_f for all points
5) Remove R_f minimum from the fixed points and place point among the displaced ones.
6) Continue until R_f is shown to be non significant

6.7.3. An approximate analysis

If there exists no covariance matrix, or only parts of them for the two epochs, an approximate analysis can be used. With this method only the elements closest to the diagonal of the co-variance matrix will be used. All unknown correlations will be set to zero. During the calculations the following steps will be taken:

1) An S-transformation, where only the sub matrixes along the diagonal are calculated, as those are the only ones needed in the rest of the calculations.
2) The test value for the Global Congruency test will be

$$T = \sum_{i=1}^{N} d_i^t (Q_{dd})_{ii}^{-1} d_i \qquad (6.27)$$

where N is the number of fixed points, $(Q_{dd})_{ii}^{-1}$ is the sub matrix for point i

This method is the same as the method of maximal displacements, only the co-variance matrix has been exchanged with an approximation.

7. Presentation techniques for deformations

This chapter deals with different ways of presenting and illustrating deformation results. In some cases, the movements of the chosen points are not the only interest, but the general movement or tilt of a certain part of an area. Since it is not practical to have a very dense network, generalisations have to be done. Two ways of doing this are to interpolate a surface in between measured points or to neglect the terrain around them. The objectives of the techniques are either to make the general tendencies easily visible or to extract other sorts of information through the deformation data, e.g. strain in the ground.

7.1. Illustration by vectors and error ellipses

This is a common method for illustration of the deformation results. The method has been used in chapter 8 for illustration of the plane displacements by the PANDA program (see GeoTec GmbH, 1992). The presentation base is a map showing the configuration of the deformed points. In the planar case, lines are attached to all points showing the approximate magnitude and direction of each point movement. The standard errors for the deformations are then symbolised by ellipses that make it possible to compare the accuracy with the movement. In the following figure point 1 has no determined movement, while point 2 appears to move. The direction of the movement can be obtained relative to the local system or the north direction.

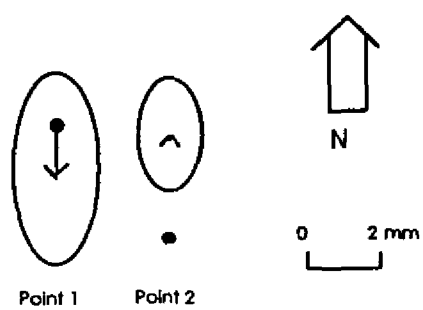

Figure 7.1. Deformation illustration by vectors and ellipses.

By studying a figure containing several points, one can get an easily grasped view of the movement situation in the tested area.

For vertical deformations, an illustrated vector can only show the magnitude of the displacement. The mean error is more difficult to illustrate, since no ellipse can be drawn. A vector showing the mean error then has to be put beside the movement vector.

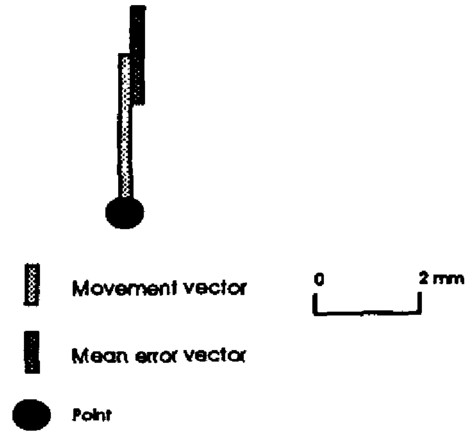

Figure 7.2. Vertical deformation illustration by vectors.

7.2. Vertical deformations illustrated by terrain models

The purpose of using terrain models to illustrate the deformations is that it makes it easy to grasp the general and local tendencies of the vertical movements. The method uses interpolation technique, so the model is only to be taken as a hypothesis. This since locations in between points obtain deformation values even though they are unknown.

For the processing a terrain model program is needed. As input data, the X and Y co-ordinates for the points are used, as well as the deformations representing Z. The program constructs a grid within the limits of given point co-ordinates. The density of the grid is entered by the user or from a default value. The program then calculates X and Y for every single line intersection in the grid. One method for calculating the Z-value of the intersection is by finding its three nearest points. This is done e.g. by using the least square method for the distance between the intersection and all given points. The three closest points (that can form a triangle enclosing the intersection) are used.

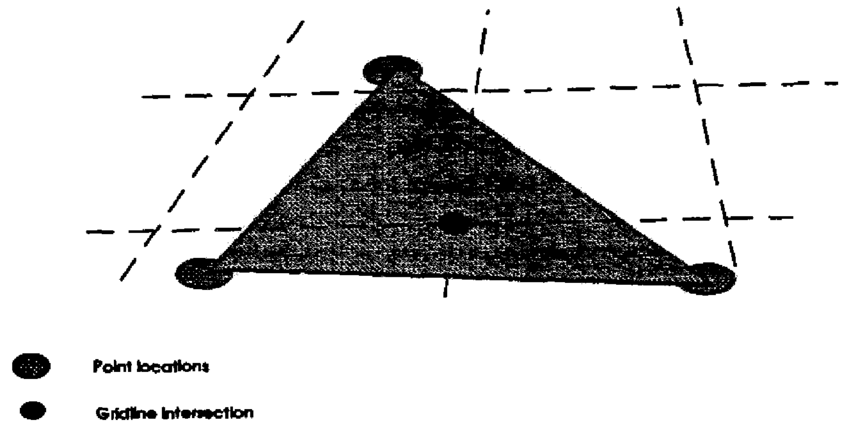

Figure 7.3. Plane creation.

A plane is constructed which involves the three points as seen in figure 7.3. The intersection then obtains the Z-value it would have if it was situated in the plane.

This method is repeated so that X, Y and Z co-ordinates are known for all intersections . By deciding the location of a certain viewpoint, a 3D ⇒2D transformation will give the intersection locations as viewed from the desired point.

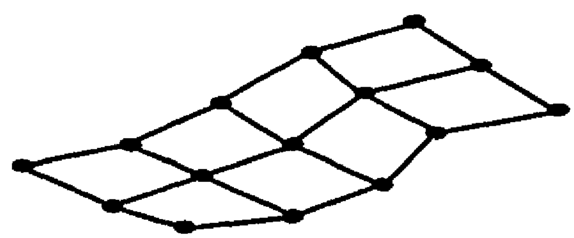
Figure 7.4. Grid construction.

The grid is then constructed by drawing the lines between all intersections. After this, graded axes may be added. An example of the whole application can be seen in figure 8.10.

7.3. Tilt changes illustrated by planes

When the positions of the points are known, it is possible to check if the measured area as a whole is tilted in any size and direction. This application is useful in cases where tilt changes are of main interest. An example of those is volcano slopes, where a tilt change may be a sign of a coming eruption. Another use is in building or road restoration, where it can be controlled if a surface (e.g. a floor) has tilted during a certain period of time.

7.3.1. Tilt calculation method described by Mori (1988)

A mathematical method is explained by Mr H Mori (see Mori and Suzuki, 1988). The method uses points in a network where the levels and plane co-ordinates have been calculated relative to a chosen reference point. The X and Y values are not needed to be very accurate, since variations only affect the planar directions slightly. Therefore, the plane co-ordinates are often looked upon as constant over the epochs.

Two observation epochs of a certain network are then compared to find the tilt change for the concerned period of time. The comparison is made by calculating tilt change and azimuth for all combinations of two points in the network. These part results can then be plotted as an early check for gross errors. An example is seen in the following figure.

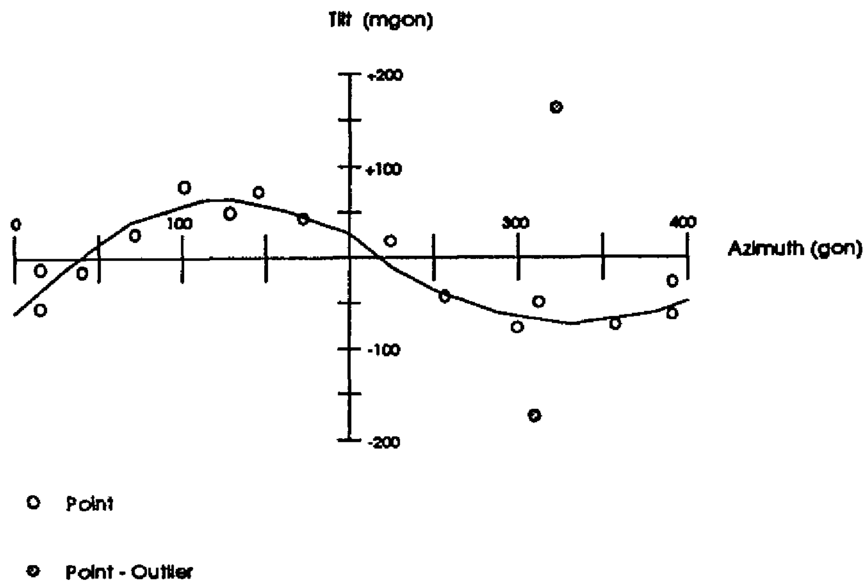

Figure 7.5. Plot of tilt changes and azimuths for all point pairs.

For a definite tilt tendency, one should be able to distinguish a sine-shaped curve with a certain phase shift. If such one is drawn, the gross errors can be detected as outliers in the plot. An approximate tilt direction can be obtained by reading the azimuth of the maximum value of the curve.

For the mathematical process, the following parameters are needed. Except for the parameter i, the unit is gon.

i	Point pair number
θ	Azimuth of point pair
A	Amplitude of tilt change vector
A_1, A_2	Part results
ω	Azimuth of tilt change vector
T_i	Observed tilt change

Table 7.1. Parameters for the Mori calculation method.

$$A_1 = \frac{(\sum_{i=1}^{n} T_i * \cos\theta_i) * (\sum_{i=1}^{n} \cos\theta_i * \sin\theta_i) - (\sum_{i=1}^{n} T_i * \sin\theta_i) * (\sum_{i=1}^{n} \cos^2\theta_i)}{(\sum_{i=1}^{n} \cos\theta_i * \sin\theta_i)^2 - (\sum_{i=1}^{n} \sin^2\theta_i) * (\sum_{i=1}^{n} \cos^2\theta_i)} \quad (7.28)$$

$$A_2 = \frac{(\sum_{i=1}^{n} T_i * \sin\theta_i) * (\sum_{i=1}^{n} \cos\theta_i * \sin\theta_i) - (\sum_{i=1}^{n} T_i * \cos\theta_i) * (\sum_{i=1}^{n} \sin^2\theta_i)}{(\sum_{i=1}^{n} \cos\theta_i * \sin\theta_i)^2 - (\sum_{i=1}^{n} \sin^2\theta_i) * (\sum_{i=1}^{n} \cos^2\theta_i)} \quad (7.29)$$

$$A = \sqrt{A_1 + A_2} \quad (7.30)$$

$$\omega = \text{Arctan}(A_1 + A_2) \quad (7.31)$$

The result is obtained as a vector with magnitude A and direction ω. The presentation form seems to be accurate and with an easily comprehensive result, but any error estimations were not found. For this reason, a more simple method which involves error estimation will be presented.

7.3.2. The Surface fitting method

The method could be described as an adjustment of a plane as a function of displacements between two epochs. The objective is to calculate the tilt of the plane which describes how the area as a whole is tilting. By using statistical methods, an idea of the significance of the tilt can be obtained.

The method was constructed by the authors by using well known adjustment and statistical methods. It should be seen as a complement to the earlier mentioned "Mori" method.

X and Y co-ordinates are determined relative to the centroid of the network, while the Z-values are represented by the deformations. A plane through the measured area is then determined by using least squares.

Through the mentioned input data, a linear expression can be determined for the plane in the following form:

$$f(x,y) = k_x x + k_y y \qquad (7.32)$$

The adjustment can be looked upon as a linear regression for the X- and Y-axis separately. The input data is represented in vector and matrix form, where A is the design matrix containing x- and y values for the points. L is the observation vector (deformations). P is a weight matrix containing the standard errors of the deformations in the diagonal, and F is a unit matrix. The vectors and matrices are represented as follows:

$$A = \begin{vmatrix} x_1 & y_1 \\ \cdot & \cdot \\ x_n & y_n \end{vmatrix} \qquad (7.33)$$

$$L = \begin{vmatrix} d_1 \\ \cdot \\ d_n \end{vmatrix} \quad \text{(d = deformation)} \qquad (7.34)$$

$$P = \begin{vmatrix} s_{d_1} & \cdots & \cdots & 0 \\ 0 & s_{d_2} & \cdots & 0 \\ \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdots & s_{d_n} \end{vmatrix} \quad (s_d = \text{deform. std. error}) \qquad (7.35)$$

The element adjustment is then performed as presented in chapter 4. The adjustment result is to be found in the following parameters:

$$X = \begin{vmatrix} k_x \\ k_y \end{vmatrix} \quad (7.36)$$

$$S_x = s_0 * \sqrt{Q_{vv_{11}}} \quad (7.37)$$

$$S_y = s_0 * \sqrt{Q_{vv_{22}}} \quad (7.38)$$

where s_x denotes the standard error in k_x , and s_y denotes the standard error in k_y. The plane equation is obtained by using the formula (7.32).

The tilt of each axis can be obtained by using the arctangence function on the k-values respectively.

By using the results of the element adjustment, a statistical test can be done to see whether the X or Y axis have moved or not. A test value from the t-distribution is obtained from a table. The parameter for the table is the degree of freedom (f) from the adjustment, in this case number of points minus two. The confidence interval is arbitrary, but 95% is often used. The test is then carried through as follows:

$$H_{x=0} \quad : \quad 0 < k_x - t_{0.025}(f) * s_x \quad (7.39)$$

$$H_{y=0} \quad : \quad 0 < k_y - t_{0.025}(f) * s_y \quad (7.40)$$

If one or both of the hypothesis are true, a tilt is proven statistically for the X- and/or Y-axis.

A tilt test of the level deformations 1990-1992 showed a tilt α of -2.05 mgon for the X-axis and +1.45 mgon for the Y-axis. Both passed the statistical test with minimum marginal since the $H_{x=0}$ test resulted in +0.224 mgon and $H_{y=0}$ in +0.339 mgon. It must be mentioned though, that tilt observations are mostly made on areas with more homogenous terrain.

Another test was made limiting the used deformations to the town centre (point 1, 2, 3, 9, 13, 15, 16, 17, 18, 20 and 21). The result showed a similar tilt but passed the statistical test with a slightly larger marginal than the tilt test containing all points.

$[\alpha_x, \alpha_y]$ = [+2.16 mgon, -1.77 mgon]

$[H_{x=0}, H_{y=0}]$ = [+0.738 mgon, 0.325 mgon]

The method can also be used for fitting a second or higher degree equation to the deformation observations. The result can then be looked upon as a surface of a certain degree describing the displacements. The second degree equation is obtained by the following formula:

$$f(x,y) = k_a x^2 + k_b xy + k_c y^2 + k_d x + k_e y \qquad (7.41)$$

The calculations are then carried through in the same way as for linear expressions (see equations 7.33 - 7.38), but with a few exceptions:

- A consists of x^2, xy, y^2, x and y
- X consists of k_a, k_b, k_c, k_d and k_e
- S_{vi} contains 5 parameters

7.4. Strain components

In some deformation studies the main interest is concentrated to the tension between points. This means e.g. how three points are moving relative to each other. The movements of the points compared to the datum of the network is of less interest. The reason for the strain component study is to see if there is any tension in the ground in a special area. The tension is developed by local ground movements in different directions. This can be more hazardous to constructions as roads and buildings than large movements in the same direction.

The strain component method creates a triangle-shaped area by using distance and angle observations of three points from two epochs (see figure 7.6). If only point co-ordinates exist, the angles and distances can be obtained from them. The method calculates a vector which shows the size and direction of the largest tension in the triangle. Another vector perpendicular to the first one illustrates the size of the orthogonal stain. Since the tension can be in two directions, i.e. stress or strain, two ways of vector illustration exist (see figure 7.7).

When networks are tested for strain components, the network is divided into triangles.

7.4.1. Strain calculation method by Terada and Miyabe (1929)

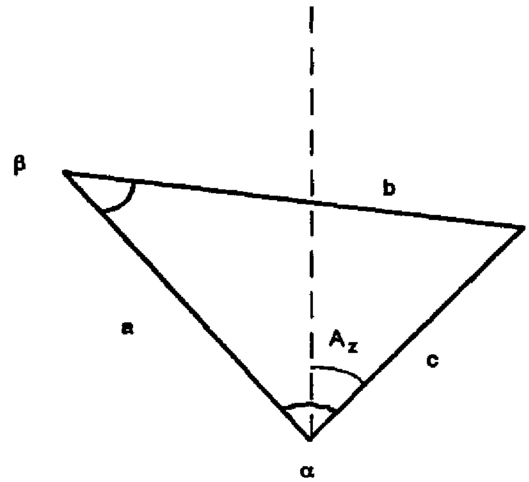

Figure 7.6. Parameters in strain component calculation.

Except for the above mentioned parameters, the distance differences between two epochs (Δa, Δb and Δc) are included. The angle differences between two epochs are neglected. The calculations are then carried through as follows:

$$\varepsilon p = \frac{\Delta a}{a} \qquad (7.42)$$

$$\varepsilon q = \frac{\Delta b}{b} \qquad (7.43)$$

$$\varepsilon r = \frac{\Delta c}{c} \qquad (7.44)$$

$$\tan\theta = \frac{(\varepsilon r - \varepsilon p)\sin^2\alpha - (\varepsilon q - \varepsilon p)\sin^2(\alpha+\beta)}{2((\varepsilon q - \varepsilon p)\sin(\alpha+\beta)\cos(\alpha+\beta) - (\varepsilon r - \varepsilon p)\sin\alpha\cos\alpha)} \qquad (7.45)$$

$$X_{\varepsilon(1\text{-}2)} = \frac{2(\varepsilon q - \dfrac{\varepsilon p(1+\cos(2(\theta+\alpha))}{1+\cos(2\theta)})}{1-\cos(2(\theta+\alpha)) - \dfrac{((1+\cos(2(\theta+\alpha))(1-\cos(2\theta))}{1+\cos(2\theta)}} \qquad (7.46)$$

$$Y_{\varepsilon(1\text{-}2)} = \frac{2\varepsilon p - (1-\cos(2\theta))\, X_{\varepsilon(1\text{-}2)}}{1+\cos(2\theta)} \qquad (7.47)$$

$$\text{If } X \geq Y \text{ then } [\varepsilon_1 = X, \varepsilon_2 = Y] \qquad (7.48)$$

$$\text{If } X < Y \text{ then } [\varepsilon_1 = Y, \varepsilon_2 = X] \qquad (7.49)$$

The results are then presented as follows:

Azimuth of the vector with largest value	:	$Dir = \theta + A_z$	(7.50)
Maximum shortening force[13]	:	$\gamma = -(\varepsilon_1 - \varepsilon_2)$	(7.51)
Aerial expansion (negative = contraction)	:	$\Delta = \varepsilon_1 + \varepsilon_2$	(7.52)

In the figure below, the dashed line symbolises stress and the perpendicular line strain.

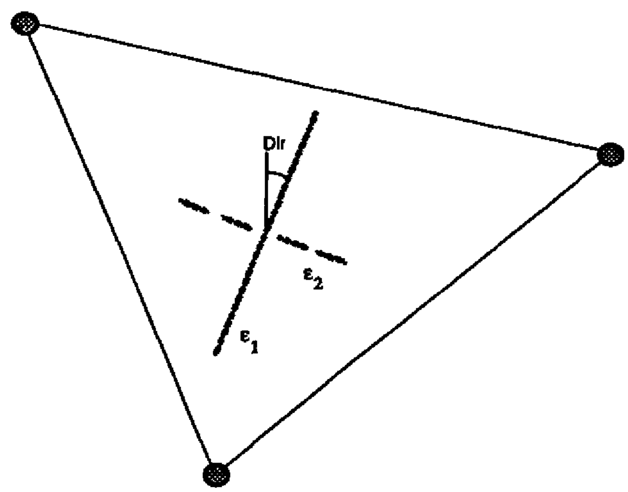

Figure 7.7. Strain component vectors.

By studying the test results, conclusions can be drawn about in which part of the network ground damages are most likely to occur. Except from the illustrations, the method gives data about the areal change of the triangle. The maximum shortening force is also received. It describes how

[13] Direct translation of the spanish term "Máximo esfuerzo cortante".

the ground gives way in a certain direction when exposed to a force in the perpendicular direction.

In some irregularly shaped networks, there may not be an ideal partition in triangles. In these cases, different constellations of triangles may be tried to see if they bring new interesting information.

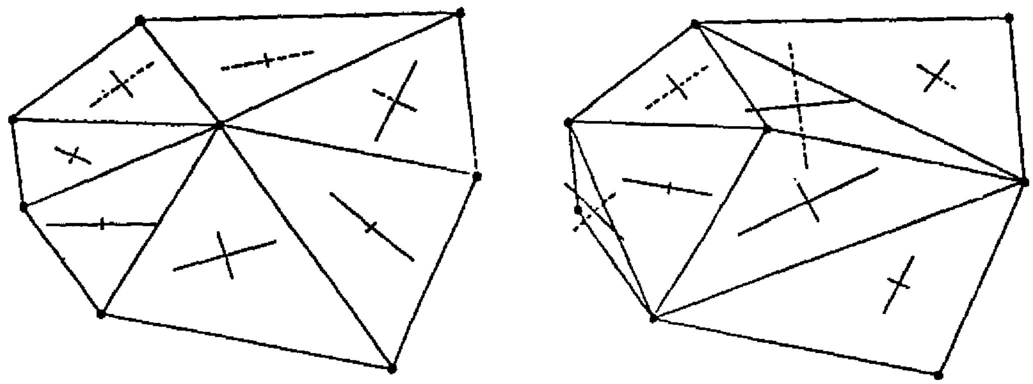

Figure 7.8. Strain components of the Irazu volcano (Van Der Laat, 1992).

The pictures above show two different constellations created from the same data. The observations are made by OVSICORI staff on a network situated on the Irazu volcano near Cartago, Costa Rica.